THERMAL SELF-FOCUSING INSTABILITY IN THE CONDUCTION LAYER OF A LASER TARGET

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Refraction is included in the stability analysis of the corona ablated from a laser target, assuming conduction restricted to a thin layer and absorption at the critical density inside it. A thermal self-focusing instability, with growth rate ~ (ion–electron collision frequency) × (electron-to-ion mass ratio), is found.

Direct-drive laser fusion is coming back at present as a possible approach to inertial confinement [1]. A major issue in this approach is whether heat conduction can efficiently smooth out unavoidable nonuniformities in the driving. Refraction and magnetic effects were recently included in the analysis of thermal smoothing, in one limit regime: extreme ablative conditions (long, moderate-intensity pulses at short wavelengths) and absorption around the critical density n_c, conduction within the corona then being restricted to a thin deflagration layer next to the target [2,3].

Here we consider the deflagration regime for uniform irradiation at normal incidence and study the stability of the known quasisteady structure of the conduction layer. Assuming both $\rho_c/\rho_{\text{target}}$ and ablated mass fraction small, the acceleration of the target is negligible, and the Rayleigh–Taylor [4] and Landau's hydrodynamical [5] instabilities can be ignored. For transverse perturbation wavenumber $k \sim$ (layer thickness)$^{-1}$, the coronal region beyond the layer need not be considered. We find that refraction makes the layer unstable for a certain k-range.

To analyse the stability of the layer we use the same equations as ref. [2], retaining time-derivatives for perturbed quantities and dropping, for simplicity, all magnetic and current terms, which proved to be unimportant for smoothing processes in the present regime:

$$\frac{\partial n_e}{\partial t} + \nabla\cdot n_e \bar{v}_i = 0 , \qquad (1)$$

$$\bar{m} n_e(\partial/\partial t + \bar{v}_i\cdot\nabla)\bar{v}_i = -\nabla(n_e T_e) , \qquad (2)$$

$$n_e T_e(\partial/\partial t + \bar{v}\cdot\nabla)\ln(T_e^{3/2}/n_e) = \nabla\cdot(\bar{K}T_e^{5/2}\nabla T_e) + I' , \qquad (3)$$

$$\nabla\cdot\bar{\tau} I = 0 , \qquad (4)$$

$$\bar{\tau}\cdot\nabla\bar{\tau} = [\nabla - \bar{\tau}(\bar{\tau}\cdot\nabla)]\ln(1-n_e/n_c)^{1/2} . \qquad (5)$$

The ion charge-number was assumed large so as to neglect ion-temperature terms: $\bar{m}$ is ρ_c/n_c, $\bar{K}T_e^{5/2}$ the heat conductivity, I' a δ-function term representing laser heating, I the local light intensity, and $\bar{\tau}$ a unit ray vector (ray-crossing may be neglected). Light is impinging from the right, and the target occupies the region $x<0$ at all times.

The zero-order quasisteady structure of the layer [2] is given by the equations

$$(\mathrm{d}/\mathrm{d}x)(n_0 v_0) = 0 ,$$

$$(\mathrm{d}/\mathrm{d}x)(\bar{m} n_0 v_0^2 + n_0 T_0) = 0 ,$$

$$(\mathrm{d}/\mathrm{d}x)[n_0 v_0(\tfrac{1}{2}\bar{m} v_0^2 + \tfrac{5}{2}T_0) - \bar{K}T_0^{5/2}\mathrm{d}T_0/\mathrm{d}x] = I_0\delta(x-x_{c0}) .$$

We just recall here some features of that structure. First, the flow velocity becomes (isothermally) sonic at the critical surface, where velocity and density gradients are singular. Second, fluid profiles are very

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smooth in the underdense part of the layer: the *minimum* temperature gradient for $x<x_{c0}$ is 24 times the *maximum* gradient for $x>x_{c0}$. Finally, at the layer exhaust (x/x_{c0} large), $n_0\to 4n_c/5$, $v_0^2\to 5T_0/3\bar{m}\to (5I_{c0}/8\rho_c)^{2/3}$.

To analyse perturbations that ripple the critical surface,

$$x_c = x_{c0} + x_{c1}\,e^{iky+\gamma t},$$

we linearize the equations in the small parameter x_{c1}/x_{c0}. In order to avoid difficulties at the density n_c we strain the x coordinate [2,3], by introducing new variables

$$s \equiv x(1-e^{iky+\gamma t}x_{c1}/x_{c0}+\ldots), \quad y=y, \quad t=t,$$

so as to locate the critical density at $s=x_{c0}$ to all orders in the expansion; we have to first order

$$I' = (1-e^{iky+\gamma t}x_{c1}/x_{c0})I_c\delta(s-x_{c0})$$

and $I_c = I_{c0}+I_{c1}\,e^{iky+\gamma t}$. We then write, for instance,

$$v_x = v_{x0}(s) + v_{x1}(s)e^{iky+\gamma t}+\ldots,$$

etc., $v_{x0}(s)$ being the same zero-order function $v_0(x)$.

Eqs. (1)–(5) lead to a seventh order system of linear differential equations for first order quantities with two eigenvalues γ and I_{c1}, requiring nine conditions (the system being homogeneous, x_{c1} may be taken arbitrary). They are:

(1)–(3) At the ablation surface ($s=0$), pressure and mass flow rate bounded, and transverse velocity continuous; equivalently, $v_{x1}=T_1=v_{y1}=0$.

(4)–(6) At the critical surface ($s=x_{c0}$), $I_1=I_{c1}$, n_1 vanishing, and other fluid variables bounded (leading to $v_{x1}-\gamma x_{c1}=T_1/2\bar{m}v_{c0}$).

(7)–(9) Far from the target (s/x_{c0} large), incoming irradiation parallel and uniform, that is, $\tau_{y1}\to 0$, $I_1\to 0$. Also, fluid variables bounded: for $R_e\gamma>0$ (instability) *one* of the fluid modes is unbounded at infinity so that a ninth boundary condition ensues.

For wavevectors such that

$$\frac{1}{100x_{c0}} \ll k \ll \frac{1}{x_{c0}}$$

an explicit approximation to the dispersion relation $\gamma(k)$ may be obtained, using (i) the large ($\sim 10^2$) zero-order ratio of overdense to underdense gradients, and (ii) the ansatz $\gamma/kv_{c0}=O(1)$. Then, for $s<x_{c0}$, we have for instance,

$$\frac{d\ln n_1}{ds} \sim \frac{d\ln n_0}{ds} \sim \frac{1}{x_{c0}} \gg k \sim \frac{\gamma}{v_{c0}},$$

and the overdense equations are immediately integrable. For $s>x_{c0}$ on the contrary, we have

$$\frac{d\ln n_1}{ds} \sim k \sim \frac{\gamma}{v_{c0}} \gg \frac{d\ln n_0}{ds} \sim \frac{1}{100x_{c0}}$$

and the underdense equations yield modes of exponential type. With an appropriate matching at the critical surface we get

$$\frac{16}{5}\left(\frac{\gamma}{v_{c0}k}\right)^6 + \frac{62}{3}\left(\frac{\gamma}{v_{c0}k}\right)^4 - \frac{75}{16}\left(\frac{\gamma}{v_{c0}k}\right)^2 - \left(\frac{25}{8}\right)^2 = 0\,. \qquad (6)$$

This equation has one unstable root satisfying the above ansatz

$$\gamma \approx 0.87 v_{c0} k\,.$$

We also carried out exact numerical computations for the dispersion relation $\gamma(k)$ assuming that γ was real and positive. (For $R_e\gamma<0$ there may be two modes unbounded as $s/x_{c0}\to +\infty$, making a total of ten boundary conditions, one too many for a nontrivial solution to exist. A Laplace transform must then be used to follow the evolution of an initial perturbation [6].) The results, in fig. 1, show perfect agreement at low kx_{c0} with eq. (6). Note that (i) starting at $kx_{c0}\approx 1.30$ there are *two* unstable roots, one vanishing just at that wavenumber and (ii) there are *none* beyond $kx_{c0}\approx 2.05$.

We may here recall that in a steady analysis ($\gamma=0$) of thermal smoothing, we had determined what driving non-uniformity $I_{\infty 1}\equiv I_1(s/x_{c0}\to +\infty)\neq 0$ was required to sustain a given nonuniformity in the absorption I_{c1}; to the accuracy of the numerical computations we had found that $I_{\infty 1}/I_{c1}=0$ at $kx_{c0}\approx 1.30$ and $I_{\infty 1}/I_{c1}>1$ at $kx_{c0}>2.05$.

Since [3]

$$x_{c0} \sim (\bar{m}/m_e)^{1/2}(T_{c0}/m_e)^{1/2}\nu_{ei}^{-1},$$

where ν_{ei} is the ion–electron collision frequency, the maximum growth rate of the instability, $\gamma\sim v_{c0}/x_{c0}$, is

$$\gamma \sim m_e\nu_{ei}/\bar{m}\,.$$

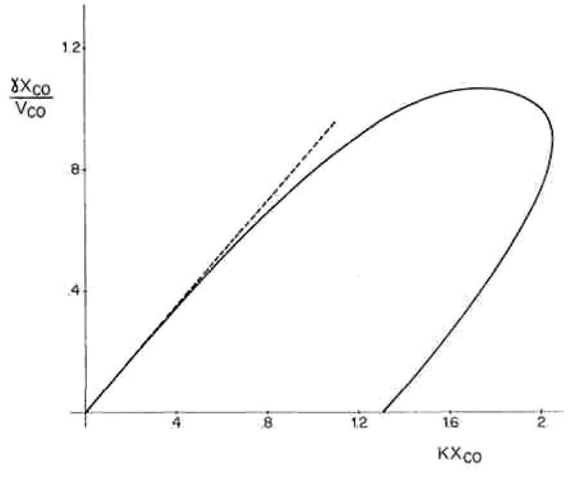

Fig. 1. Growth rate γ of instability versus transverse perturbation wavenumber k; v_{c0} and x_{c0} are unperturbed velocity at critical and distance between target and critical surfaces respectively.

The instability is caused by thermal (overall) self-focusing. Let be a ripple perturbation at some given time, and let y_+ be a y-range for which an excess intensity reaches the critical surface. In this range the near underdense plasma will refract light off y_+ (the critical surface will be here hotter and farther from the target, density and temperature transverse gradients both pointing into y_+). However, if lateral heat conduction is poor (i.e., for small wavenumber) pressure will become transversely uniform far ahead of temperature, resulting in a long-lasting, inverted density gradient in the far away plasma which refracts light into y_+, enhancing the initial perturbation. It is not yet clear why this enhancement is effective beyond $kx_{c0} \approx 1.30$ (for smaller wavenumber we had found $I_{\infty 1}/I'_{c1} < 0$ in ref. [2]).

For $k \times$ (thickness of conduction layer) small, density perturbations may reach beyond the layer, and our analysis fails; this occurs if $k(x_{c0}L)^{1/2} < O(1)$ where L is the overall coronal length [2]. For a spherical target we have $L \sim$ radius R; work in progress considers the stability of the entire corona for $k \sim R^{-1} \ll (x_{c0}R)^{-1/2}$. Recently, Coggeshall et al. [7] carried out time-dependent numerical simulations at the opposite regime: k^{-1} small compared with the conduction length, which was also the overall length (high-intensity pulse), for an imposed intensity non-uniformity.

Our results are somewhat unrealistic in neglecting inverse bremsstrahlung in a deflagration regime (long, low intensity, short wavelength pulses and large targets or focal spots). We are at present either including inverse bremsstrahlung or allowing for conduction throughout the corona.

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